Reducible to Variables Separated From (R.V.S.F.)

Differential equation of the first order cannot be solved directly by variable separable method. But by some substitution, we can reduce it to a differential equation with separable variable. Let the differential equation is of the form

$$\frac{dy}{dx} = f(ax + by + c)$$

Can be reduced to variable separable form by the substitution $ax + by + c = v$

Then

$$a + b\frac{dy}{dx} = \frac{dv}{dx}$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{b}(\frac{dv}{dx} - a)$$

Then the equation becomes

$$\frac{1}{b}\left(\frac{dv}{dx} - a\right) = f(v)$$
$$\Rightarrow \frac{dv}{dx} = a + bf(v)$$

In which the variables are separable.

Solve $\frac{dy}{dx} = (4x + y + 1)^2$ Put $4x + y + 1 = v$ $\Rightarrow 4 + \frac{dy}{dx} = \frac{dv}{dx}$ $\Rightarrow \frac{dy}{dx} = \frac{dv}{dx} - 4$ Then the given DE becomes $\frac{dv}{dx} - 4 = v^2$	$\Rightarrow \frac{dv}{dx} = V^2 + 4$ $\Rightarrow \frac{dv}{V^2+4} = dx$ [Which is reducible to variable separable form] Now integrate this we get $\int \frac{dv}{V^2+4} = \int dx$ $\Rightarrow \frac{1}{2}\tan^{-1}\frac{v}{2} = x + c$ $\Rightarrow \frac{1}{2}\tan^{-1}\frac{4x+y+1}{2} = x + c$ This is the required solution of the given DE.

Exercise: solve the following DE using RVSF

(i) $\frac{dy}{dx} = \sin(x + y) + \cos(x + y)$ (ii) $(x - y)^2\frac{dy}{dx} = a^2$ (iii) $(x + y)^2\frac{dy}{dx} = a^2$ (iv) $\frac{dy}{dx} = \sec(x + y)$	(v) $\left(\frac{x+y-a}{x+y-b}\right)\frac{dy}{dx} = \left(\frac{x+y+a}{x+y+b}\right)$ (vi) $\frac{dy}{dx} = (x + y)^2$ (vii) $\frac{dy}{dx} = (3x + y + 4)^2$